# Myth or fact? You can increase the force of your solenoid by removing turns. (Part 1 of 3)

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**Abstract**
This article examines the common claim that the strength of a solenoid can be increased by removing some turns from the coil. In the process we look at formulations for inductance, power, energy, and force. The end result clarifies the roles of several solenoid parameters, and the effect of the number of turns may be surprising to many readers. Those who would prefer to skip the mathematical details can find a qualitative explanation in the conclusion.

**Introduction**
The primary motivation for this article was the desire for students in an embedded systems course to have an understanding of solenoids sufficient to answer questions such as these:

1. You have a base solenoid design but would like to obtain more force. What parameters can you vary in order to do so?

2. You have more force than you need from a particular solenoid. What can you do to reduce it and save some electrical power in the process?

On the surface these would seem to be questions that should require only a rudimentary understanding of solenoids, but an examination of textbook literature and website coverage shows that to not be the case. For example, undergraduate textbooks on electromagnetism universally provide the following approximation to the magnetic field of a long cylindrical coil (Reference 1):

$$B = \frac{\mu N I}{l} \tag{1}$$

Here $N/l$ is the density of turns, $I$ is the current through the coil, and $\mu$ is the permeability of the core. The analysis of the field for cylindrical coils does not go much beyond that, even in textbooks oriented towards engineering students. A novice is likely to look at this equation and conclude that one can increase the magnetic field, and thus the force of the solenoid, by adding turns (not subtracting them, as posed in the title of this article). A problem with that conclusion is that practical circuits rarely drive coils with constant current; they almost always apply constant voltage.

As a second example, standard texts are likely to show the following formulations for the energy of an inductor coil (Reference 1):

$$W = \frac{1}{2} L I^2 \tag{2}$$

$$w = \frac{B^2}{2\mu} \tag{3}$$

Where *L* is the inductance, (2) is total energy, and (3) is energy density, which must be integrated over an enclosing volume to get energy. If we substitute (1) into (3) we see that both of these formula are again expressed in terms of current.

You may be thinking: "I know Ohm's law, so I will simply substitute $V = I/R$ into either of these to get an expression in terms of voltage." Not so fast. These devices are inductive, and so there is also the reactive component of impedance to consider. This cannot be ignored even if you're driving a coil with a DC waveform, because determining an inductor's reaction to a sudden change of voltage requires consideration of reactance.

Those familiar with basic electrical engineering may be tempted at this point to say: "No problem, reactance creates a phase shift between voltage and current that I don't care about, so I'll just use the magnitudes and make the following substitution:

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{2\pi f L} \tag{4}$$

where $f$ is frequency. If we use radial frequency, $\omega = 2\pi f$, drop the magnitude symbol, and substitute into (2) we get:

$$W = \frac{V^2}{2\omega^2 L} \tag{5}$$

This same expression can be obtained by multiplying the average reactive power in an inductor by one radial time period, $1/\omega$, in order to obtain the energy over one cycle:

$$P_{ave} = V_{rms} I_{rms} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} = \frac{V^2}{2\omega L} \tag{6}$$

$$W = P_{ave} T = P_{ave} \frac{1}{\omega} = \frac{V^2}{2\omega^2 L} \tag{7}$$

In this case, we can simplify the development by normalizing, at least temporarily, to a radial frequency of 1 rad/sec, leading to:

$$W = \frac{V^2}{2L} \tag{8}$$

This has a nice symmetry to (2) for those who are familiar with the power delivered to a resistor. This also appears to give us what we are seeking: an expression for the energy in a coil for a constant voltage. Of course we assumed AC voltage to get here, but perhaps that is acceptable since we all know that a solenoid that works for AC can be made to work for DC. For any readers that are wondering how a solenoid works for AC when the *B* field is constantly reversing, all you need to realize is that the force on the armature will be in the direction that increases inductance. Thus, regardless of the direction of *B*, the armature will always be pulled into the coil.

Unfortunately, pursuing the formulation in (8) leads to a result that does not approximate real solenoids. That analysis, along with an explanation of the mismatch, is given in **Appendix A**. For now, it will prove more useful to pursue an approach that does work, starting from a realistic equivalent circuit that includes both an ideal inductor along with a series resistance to represent copper losses in the coil.

**Equivalent Circuit**

**Figure 1** shows an equivalent circuit of an ideal inductor (the coil of the solenoid), along with a series resistor representing the lumped resistive losses of the coil. The switch is thrown at time *t*=0, and we wish to determine *I(t)* and *V(t)* across the inductor. The product of those will give us power, and we will integrate the power to get energy. For purposes of answering the turns question, it may be sufficient to look at the energy with the armature at one or more arbitrary positions.

The positions of armature out and armature all the way in are the easiest to analyze because the inductance is easy to estimate in those positions. However, it is also instructive to differentiate energy with respect to armature position in order to get force, and we can then compare the force vs. position to force-stroke curves for real solenoids. This does, in fact, provide a justification for rejecting a model based on (8).

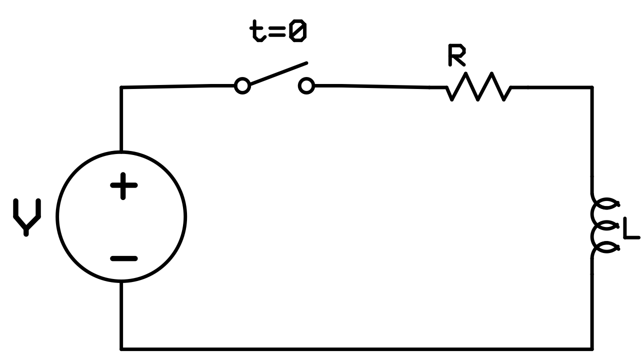


*Figure 1: Equivalent Circuit*

A detailed analysis of the time-varying current and inductor voltage can be found in any electric circuit textbook (Reference 2) and at many sites on the web (Reference 3). The transient response of the inductor is required here, and may be obtained using Laplace circuit analysis or via a straightforward solution to a first-order differential equation. Here we just make use of the pertinent results, rather than cover the details.

The voltage across, and current through, the inductor are:

$$V_L = Ve^{-\frac{t}{\tau}} \tag{9}$$

$$I_L = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad (10)$$

Where $\tau = L/R$ is called the *RL* time constant. The waveforms are shown in **Figure 2** and **Figure 3**. Note that the current builds gradually to $V/R$ and the voltage spikes to $V$ and then falls off gradually to zero. The circuit comes to within ≈37% of its final state in $\tau$ seconds.

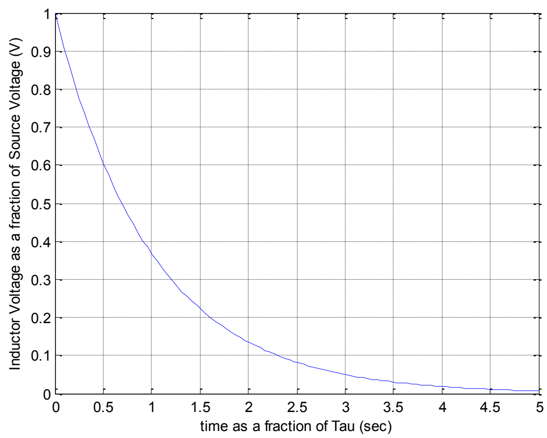


*Figure 2: Voltage across the inductor*

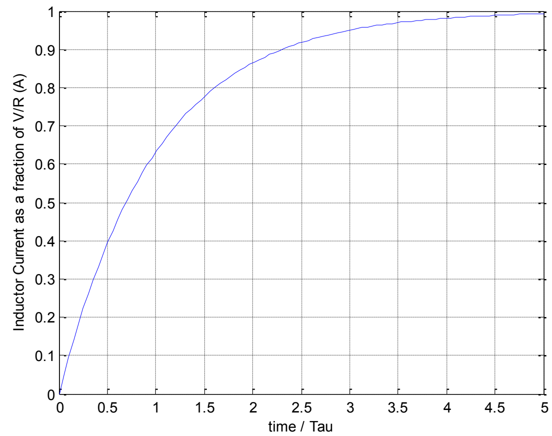


*Figure 3: Current through the inductor*

Multiplying voltage and current gives power:

$$P_L = I_L V_L = \frac{V^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right) \qquad (11)$$

Integrating the power gives the energy required to bring the inductor up to its steady-state current:

$$\begin{aligned} W_L &= \int_0^\infty P_L\,dt \\ &= \int_0^\infty \frac{V^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right)dt \\ &= \frac{V^2}{R}\left[-\tau e^{-\frac{t}{\tau}} + \frac{\tau}{2}e^{-\frac{2t}{\tau}}\right]_0^\infty \\ &= \frac{V^2}{R}\left[\tau - \frac{\tau}{2}\right] = \frac{V^2}{R}\frac{\tau}{2} = \frac{V^2 L}{2R^2} \end{aligned} \qquad (12)$$

Like (8), this equation for energy gives us what we desire: an expression in terms of voltage instead of current. Note, however, that there are two big differences: (a) it includes resistance, which better represents the real world, and (b) inductance appears in the numerator instead of the denominator.

**About the author**

***Paul H. Schimpf*** holds the B.S., M.S., and Ph.D. degrees in Electrical Engineering from the University of Washington, with respective degree specialties in digital electronics, embedded signal processing, and modeling electromagnetic fields in complex domains such as the human body.

He previously worked in the electronics industry for 13 years at Honeywell Marine Systems in Mukilteo Washington (which later became a division of Alliant Techsystems, Hughes Aerospace, and Raytheon), where he attained the level of Senior Principal Development Engineer, prior to entering a career in academics. He was previously a tenured Associate Professor of Electrical Engineering and Computer Science at Washington State University, and is presently Professor and Chair of Computer Science at Eastern Washington University.

One of his hobbies is the restoration and repair of arcade games, of which he presently owns three: a Chicago Coin Twin Rifle (electro-mechanical), a Stern Meteor (early solid state), and a Williams Fish Tales (late solid state). Paul recently designed a printed circuit board to interface an Arduino Mega to 85 different Bally/Stern pinball machines so that his embedded systems students can program pinball game logic using a modern processor. The board includes the capability to play sound effects from WAV files stored on an SD card.